OKLAHOMA STATE UNIVERSITY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 System Theory
Fall 1997
Final Exam

Name : ______________________________

Student ID: ________________________________

E-Mail Address:_______________________________________

Problem 1:

Find the *observable* canonical form realization (in minimal order) from continuous-time system

$$\frac{d^4y(t)}{dt^4}+3t\frac{d^3y(t)}{dt^3}+4\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+\alpha(t)y(t)=\frac{d^2u(t)}{dt^2}+e^{-t}\frac{du(t)}{dt}+u(t)\,.$$

Notice that gain blocks may be *time* dependent.

Problem 2:

Show that if λ is an eigenvalue of a matrix A with corresponding eigenvector v, then $f(\lambda)$ is an eigenvalue of the matrix function $f(A)$ with the same eigenvector v.

(Hint: given $Av = \lambda v$ and show $f(A)v = f(\lambda)v$)

Problem 3:
Let

$$C = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix},$$

find a matrix B and existing condition, such that $e^B = C$. Is it true that for any nonsingular matrix C, there exists a matrix B such that $e^B = C$. Justify your answer.

Problem 4:

For the Jordan block given by

$$J = \begin{bmatrix} \lambda & 1 & 0 & \cdots & 0 \\ 0 & \lambda & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & \lambda \end{bmatrix},$$

where $J \in \Re^{t \times t}$, show that

$$J^k = \begin{bmatrix} \lambda^k & k\lambda^{k-1} & \frac{k(k-1)}{2!}\lambda^{k-2} & \cdots & \frac{k(k-1)\cdots(k-t+2)}{(t-1)!}\lambda^{k-(t-1)} \\ 0 & \lambda^k & k\lambda^{k-1} & \cdots & \frac{k(k-1)\cdots(k-t+1)}{(t-2)!}\lambda^{k-(t-2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & k\lambda^{k-1} \\ 0 & 0 & 0 & \cdots & \lambda^k \end{bmatrix},$$

where $k \geq t-1$.